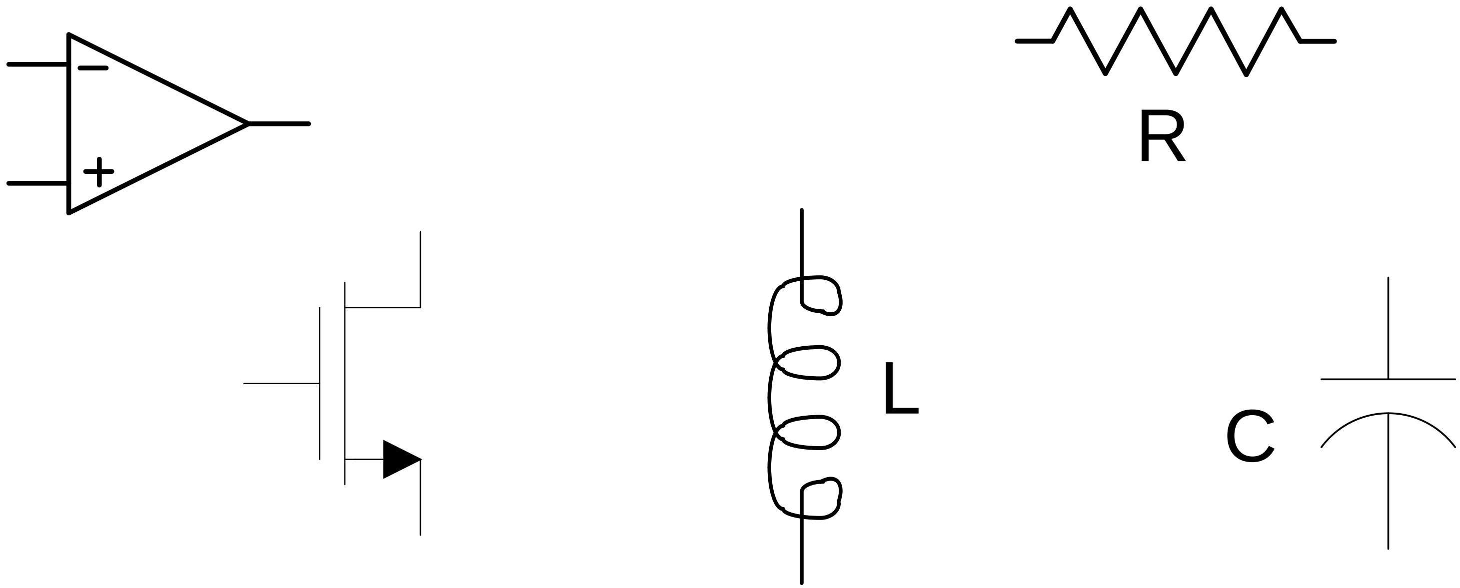
EE 508
Lecture 13

Statistical Characterization of Filter Characteristics

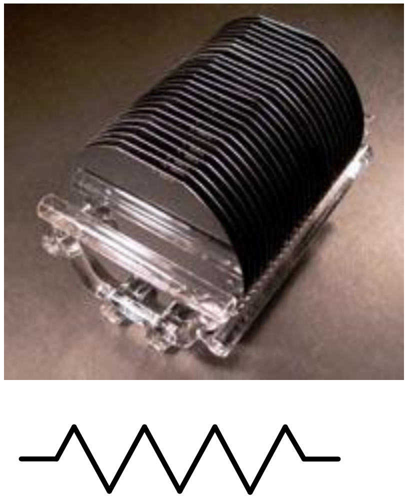
Components used to build filters are not precisely predictable



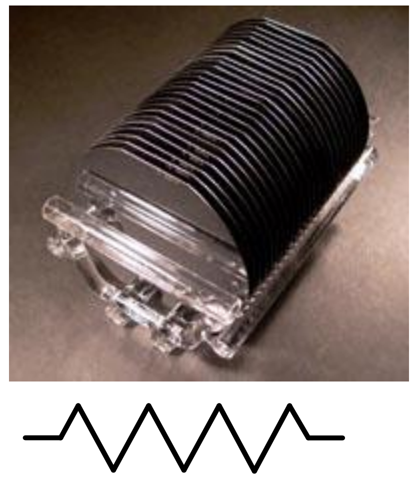
- Temperature Variations
- Manufacturing Variations
- Aging
- Model variations

- Different approaches are used to address each of these problems
- Manufacturing variations is one of the most challenging problems for building integrate filters and will be the focus of this lecture

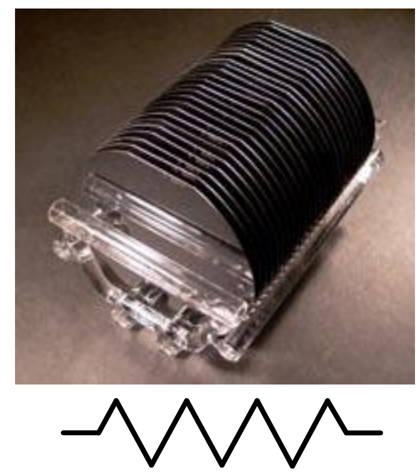
Wafers are processed in “batches” or “lots” of 20 to 40 wafers and variations occur over time (process not completely stationary) and over location



$R(t_1)$



$R(t_2)$



$R(t_3)$

These variations are often the major contributor to process variability and can be in the $\pm 30\%$ range or larger

These variations often look like random variations

Within a batch, individual wafers are subjected to some variability during processing

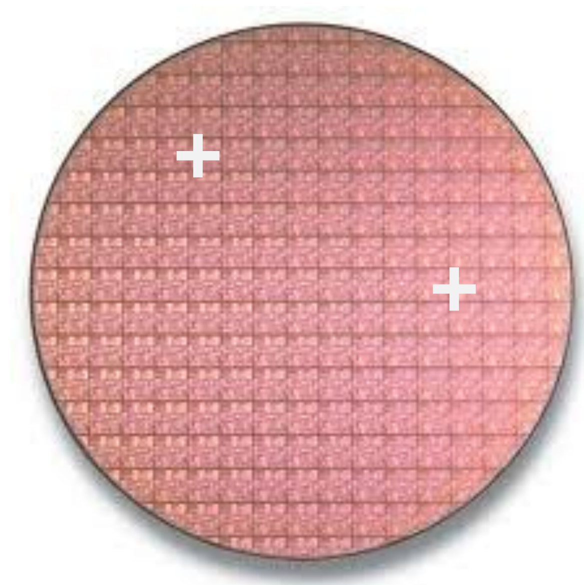
Temperature may vary with position of wafer in the boat during diffusion

Environment may vary with position of wafer in boat during diffusion or other processing steps

This variation causes characteristics of components to vary from wafer-to-wafer

These variations often look like random variations

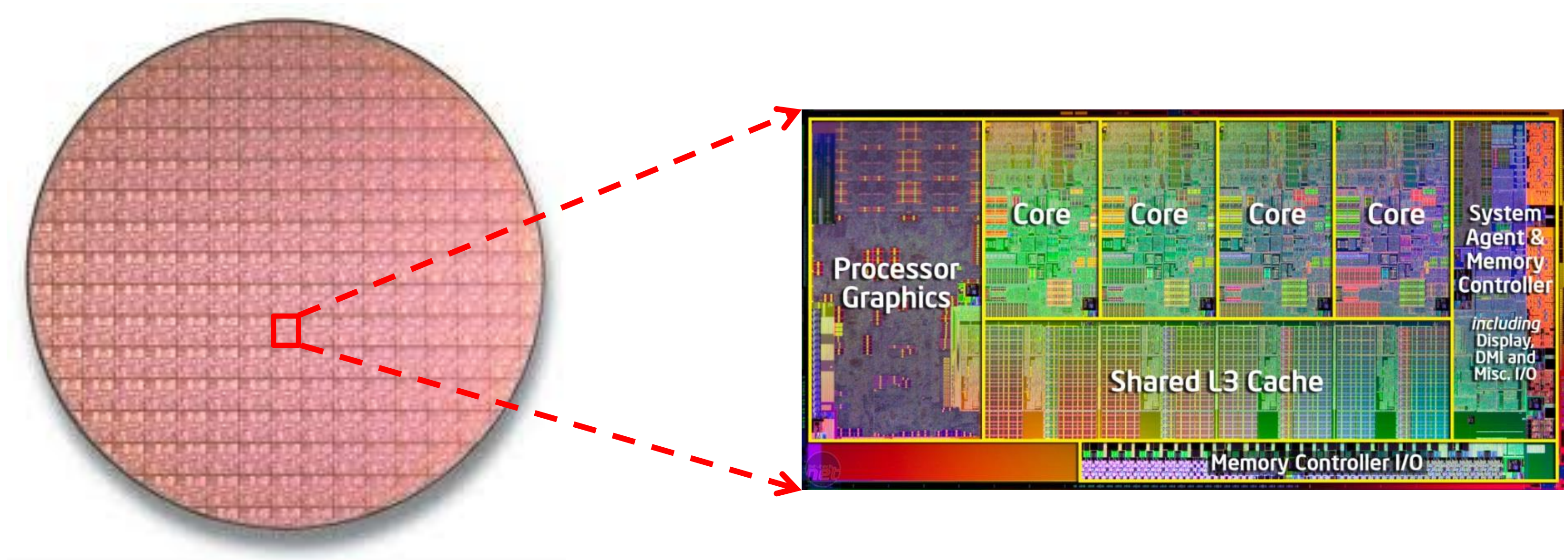
Environment may vary across individual wafers due to gradients in environmental variables during processing



This variation causes characteristics of components to vary from die to die on a wafer

These variations often look like random variations

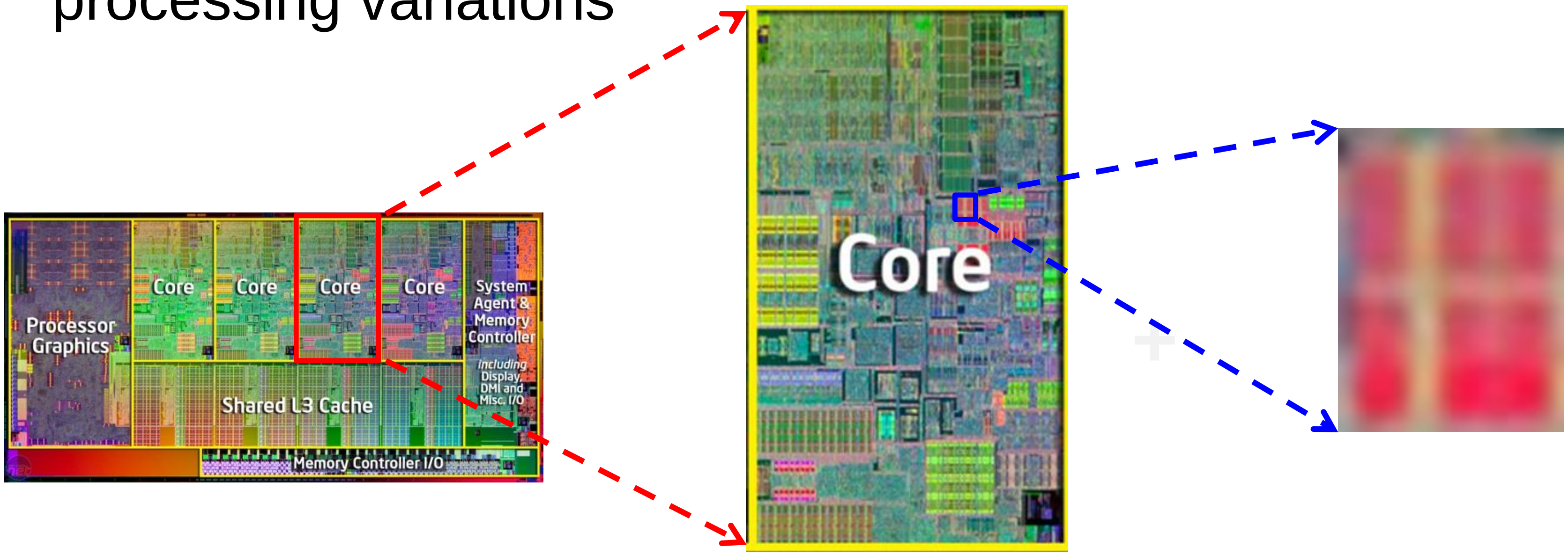
Smaller variations may occur across individual die due to gradients in environmental variables during processing



This variation causes characteristics of components to vary across a die

These variations often look like random variations

Even smaller variations may occur across individual closely placed devices due to local gradients and local random processing variations

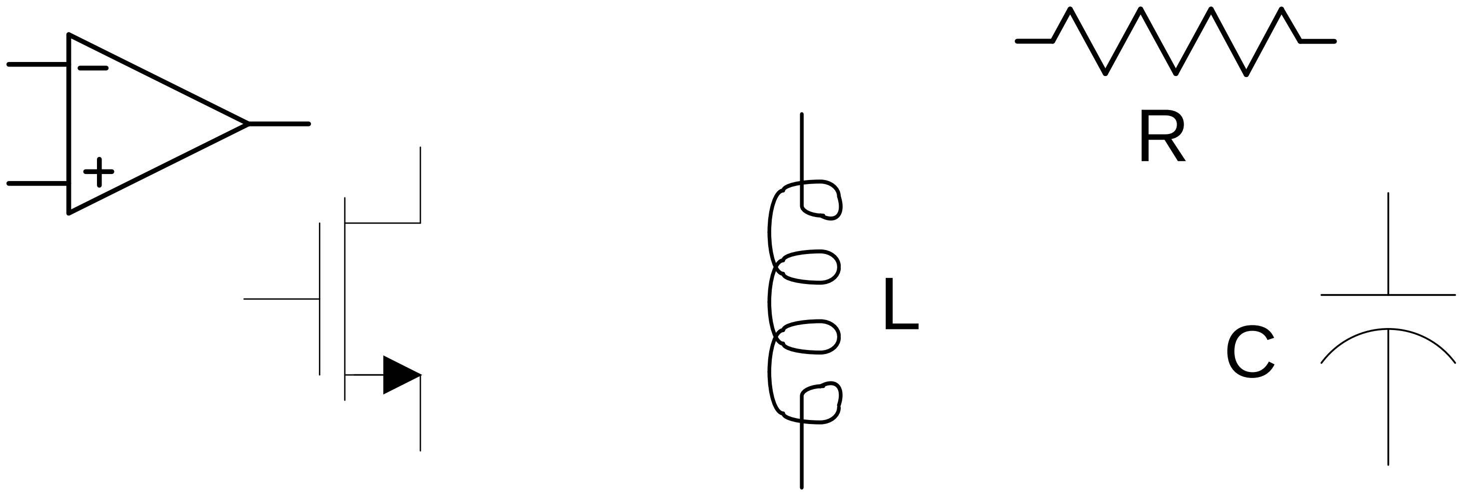


This variation introduces local gradients in device characteristics as well as local random variations

The direction and magnitude of the local gradients are random variables

The local random variations are also random variables

Effects of manufacturing variations on components



- A rigorous statistical analysis can be used to analytically predict how components vary and how component variations impact circuit performance

- Montecarlo simulations are often used to simulate effects of component variations
  - Requires minimal statistical knowledge to use MC simulations
  - Simulation times may be prohibitively long to get useful results
  - Gives little insight into specific source of problems
  - Must be sure to correctly include correlations in setup
- Often key statistical information is not readily available from the foundry

Modeling process variations in semiconductor processes

R

$$X = X_{NOM} + x_{RPROC} + x_{RWAFER} + x_{RDIE} + x_{RLGRAD} + x_{RLVAR}$$

X_{NOM} is the nominal value of the parameter (typically TT) and is a constant and part of the standard device model

x_{RPROC} is a random variable that changes from one “lot” of wafers to another

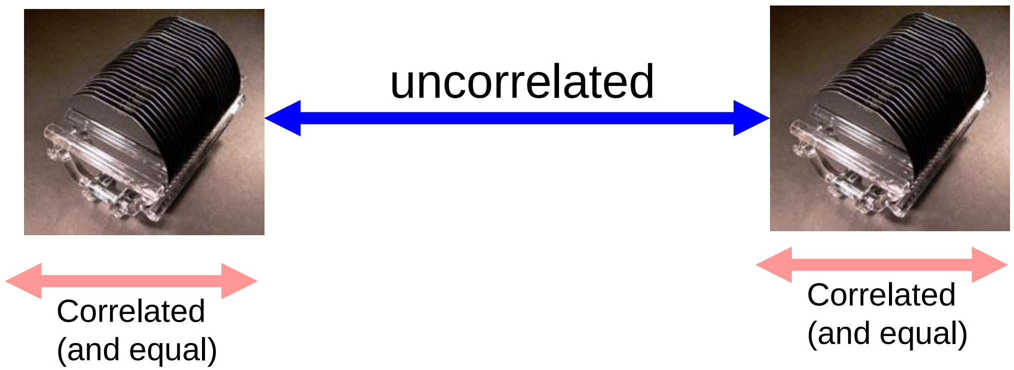
x_{RWAFER} is a random variable that changes from one wafer to another in a batch

x_{RDIE} is a random variable that changes from die to another on a wafer

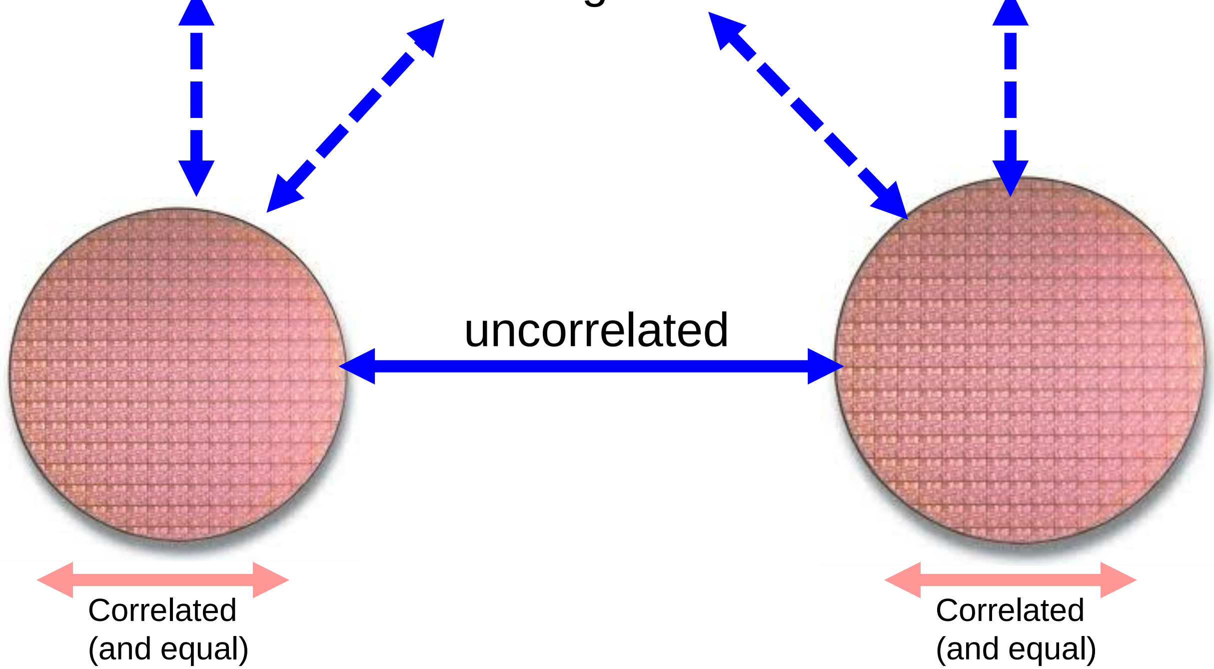
x_{RLGRAD} is a random variable that is comprised of a magnitude and direction which are themselves both random variables and characterizes very local variations on a die

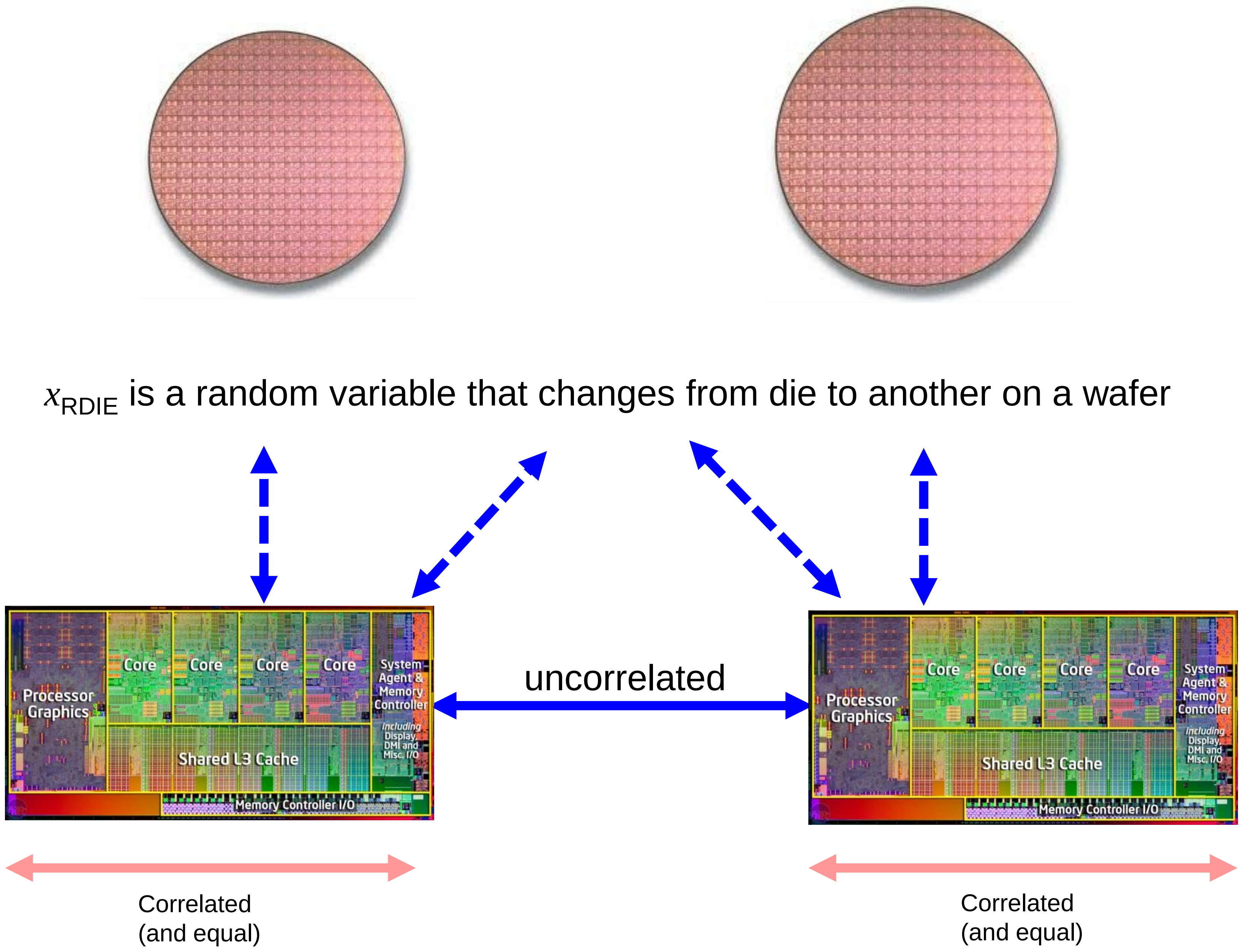
x_{RLVAR} is a random variable that characterizes very local variations on a die

x_{RPROC} is a random variable that changes from one “lot” of wafers to another

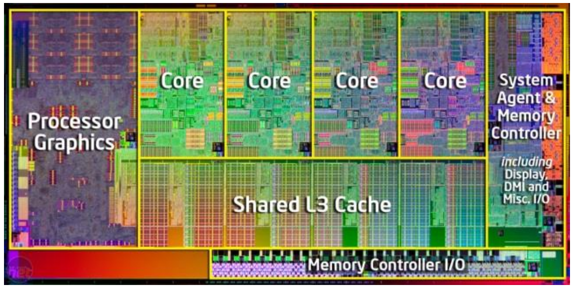


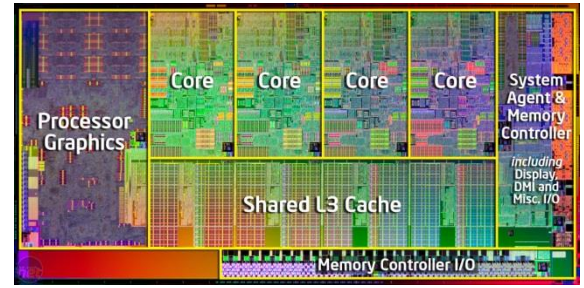
x_{RWAFER} is a random variable that changes from one wafer to another in a batch





x_{RDIE} is a random variable that changes from die to another on a wafer
uncorrelated
Core
Core
Core
Core
Processor Graphics
System Agent & Memory Controller
Including Display, DMI and Misc. I/O
Shared L3 Cache
Memory Controller I/O
Correlated (and equal)
Correlated (and equal)





x_{RLGRAD} is a random variable that is comprised of a magnitude and direction which are themselves both random variables and characterizes very local variations on a die

x_{RLVAR} is a random variable that characterizes very local variations on a die

uncorrelated

x_{RLGRAD} x_{RLVAR}

X_{RLGRAD} Correlated (and equal)

X_{RLVAR} Uncorrelated

X_{RLGRAD} Correlated (and equal)

X_{RLVAR} Uncorrelated

Modeling process variations in semiconductor processes

R

$$X = X_{NOM} + x_{RPROC} + x_{RWAFER} + x_{RDIE} + x_{RLGRAD} + x_{RLVAR}$$

x_{RPROC}, x_{RWAFER}, x_{RDIE}, x_{RLVAR} often assumed to be Gausian with zero mean

Magnitude of x_{RLGRAD} is usually assumed Gaussian with zero mean, direction is uniform from 0º to 360º

$$\sigma_{PROC} >> \sigma_{WAFER} >> \sigma_{DIE}$$

$$\sigma_{DIE} >> \sigma_{LVAR}$$

$$\sigma_{DIE} >> \sigma_{|GRAD|}$$

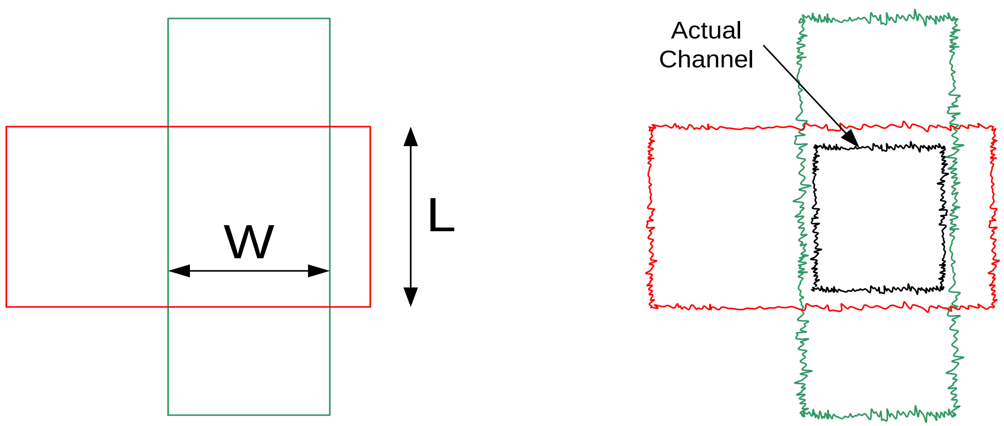
σ_{LVAR} Strongly dependent upon area and layout

$$\sigma_{LVAR} \cong \frac{1}{\sqrt{\text{Area}}}$$

$$\sigma_{LVAR} \cong \text{Perimeter}$$

Relative size between σ_{LVAR} and $\sigma_{|GRAD|}$ dependent upon A, P, and process

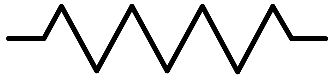
Effects of layout on local random variations



Drawn and Actual Features for MOS Transistor

Variations also occur vertically in both oxide thickness and doping levels/profiles and often these will dominate the lateral effects

Modeling process variations in semiconductor processes



R

- Statistics associated with value of dimensioned parameters (poles, GB, SR,R,C,transresistance gains, transconductance gains, … dominated by x_{RPROC})

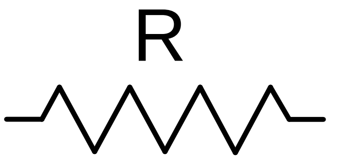
- Statistics associated with matching/sensitive dimensionless parameters such as voltage or current gains, component ratios, pole Q, … (almost always closely placed) dominated by x_{RLGRAD} and x_{RLVAR} (because locally x_{RPROC}, x_{RWAFER}, x_{RDIE} are all correlated and equal)

- Gradients are dominantly linear if spacing is not too large

- Special layout techniques using common centroid approaches can be used to eliminate (or dramatically reduce) linear gradient effects so, if employed, matching/sensitive parameters dominated by x_{RLVAR} but occasionally common centroid layouts become impractical or areas become too large so that gradients become nonlinear and in these cases gradient effects will still limit performance

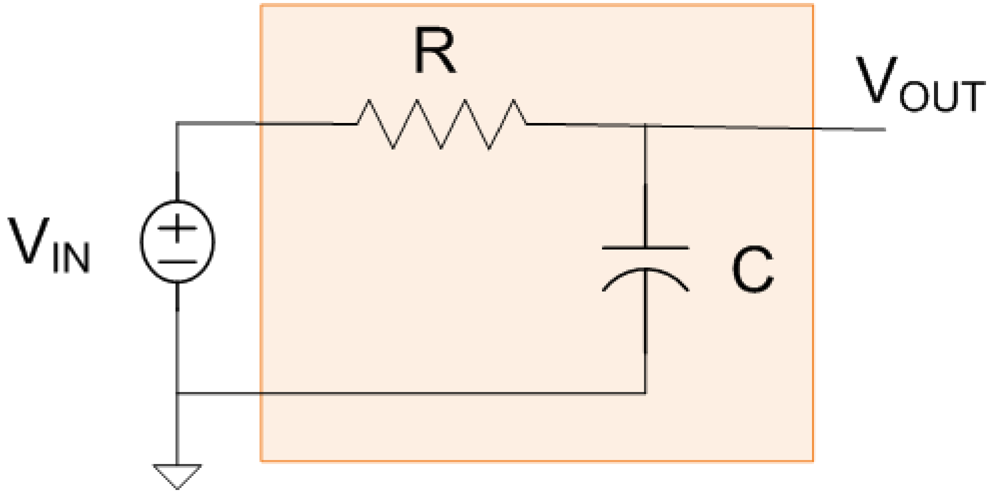
- Higher-order gradient effects can be eliminated with layout approaches that cancel higher “moments” but area and effort may not be attractive

Be sure correct statistical information is available when doing a statistical analysis using either analytical or Montecarlo methods



- Some statistics associated with making many measurements over many devices over many lots of wafers
- Some statistics associated with many measurements in a particular process run
- Some statistics associated with making many measurements across a wafer
- Some statistics associated with making many measurements on closely-placed devices
- Some statistics associated with making many measurements on closely-placed devices that have common-centroid layouts
- Some statistics presented (particularly in literature or occasionally in PDK) with limited information about how data was gathered

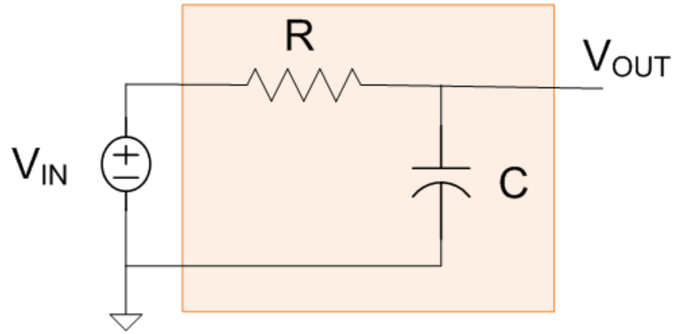
Statistical Modeling of dimensioned parameters - example



Determine the standard deviation of the pole frequency (or band edge) of the first-order passive filter.

Assume the process variables are zero mean with standard deviations given by

$$\sigma_{\frac{R_{PROC}}{R_{NOM}}} = 0.2 \qquad \sigma_{\frac{C_{PROC}}{C_{NOM}}} = 0.1$$



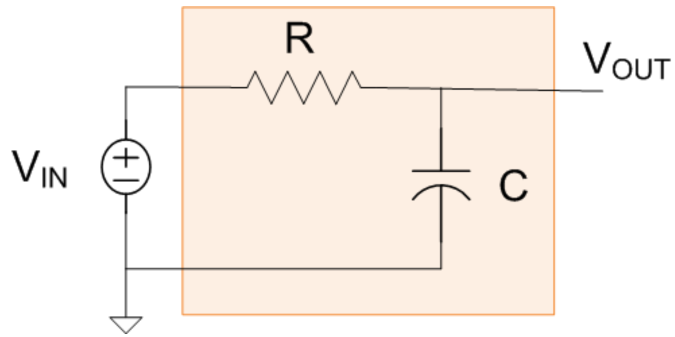
$$p = \frac{1}{RC}$$

Since R and C are random variables, the pole p is also a random variable

Theorem: The sum of uncorrelated Gaussian random variables is a multivariate Gaussian random variable

Theorem: If $X_1 \ldots X_m$ are uncorrelated random variables with standard deviations $\sigma_1, \sigma_2, \ldots \sigma_m$, and $a_1, a_2, \ldots a_m$ are constants, then the standard deviation of the random variable $y = \sum_{i=1}^{m} a_i X_i$ is given by the expression

$$\sigma_y = \sqrt{\sum_{i=1}^{m} a_i^2 \sigma_i^2}$$



$$p = \frac{1}{RC}$$

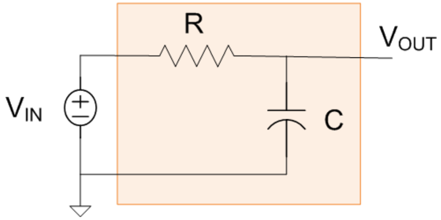
Since R and C are random variables, the pole p is also a random variable

$$p = \frac{1}{(\mathrm{R}_{\mathrm{NOM}} + R_{RAN})(\mathrm{C}_{\mathrm{NOM}} + C_{RAN})}$$

Unfortunately the pdf p which is the reciprocal of the product of Gaussian variables is very difficult to obtain

Observe can express p as

$$p = \frac{1}{(\mathrm{R}_{\mathrm{NOM}} + R_{RAN})(\mathrm{C}_{\mathrm{NOM}} + C_{RAN})} = \left(\frac{1}{\mathrm{R}_{\mathrm{NOM}}\mathrm{C}_{\mathrm{NOM}}}\right)\left(\frac{1}{\left[1+\frac{R_{RAN}}{\mathrm{R}_{\mathrm{NOM}}}\right]\left[1+\frac{C_{RAN}}{\mathrm{C}_{\mathrm{NOM}}}\right]}\right)$$



$$p = \frac{1}{RC}$$

$$p = \frac{1}{(R_{NOM}+R_{RAN})(C_{NOM}+C_{RAN})} = \left(\frac{1}{R_{NOM}C_{NOM}}\right)\left(\frac{1}{\left[1+\frac{R_{RAN}}{R_{NOM}}\right]\left[1+\frac{C_{RAN}}{C_{NOM}}\right]}\right)$$

But $R_{RAN} << R_{NOM}$ and $C_{RAN} << C_{NOM}$

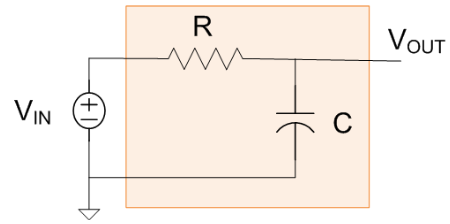
It thus follows from a truncated power series expansion of the two-variable fraction that

$$p \simeq \left(\frac{1}{R_{NOM}C_{NOM}}\right)\left(\left[1-\frac{R_{RAN}}{R_{NOM}}\right]\left[1-\frac{C_{RAN}}{C_{NOM}}\right]\right)$$

$$p \simeq \left(\frac{1}{R_{NOM}C_{NOM}}\right)\left(1-\frac{R_{RAN}}{R_{NOM}}-\frac{C_{RAN}}{C_{NOM}}\right)$$

These operations were used to linearize p in terms of the random variables !

Note that p is the sum of two Gaussian random variables that are assumed to be uncorrelated so p is also Gaussian



$$p = \frac{1}{RC}$$

$$p \simeq \left(\frac{1}{R_{NOM}C_{NOM}}\right)\left(1 - \frac{R_{RAN}}{R_{NOM}} - \frac{C_{RAN}}{C_{NOM}}\right)$$

It thus follows from the theorem that

$$\sigma_p \simeq \left(\frac{1}{R_{NOM}C_{NOM}}\right)\sqrt{\sigma^2_{\frac{R_{RAN}}{R_{NOM}}} + \sigma^2_{\frac{C_{RAN}}{C_{NOM}}}}$$

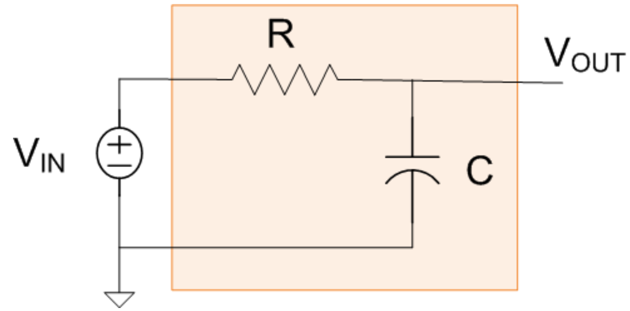
But the nominal value of the pole is $p_{NOM} \simeq \dfrac{1}{R_{NOM}C_{NOM}}$

It thus follows that

$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{\sigma^2_{\frac{R_{RAN}}{R_{NOM}}} + \sigma^2_{\frac{C_{RAN}}{C_{NOM}}}}$$

Observe:

$$\frac{p}{p_{NOM}} \sim N\left(1, \sigma_{\frac{p}{p_{NOM}}}\right)$$



$$p = \frac{1}{RC}$$

$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{\sigma^2_{\frac{R_{RAN}}{R_{NOM}}} + \sigma^2_{\frac{C_{RAN}}{C_{NOM}}}}$$

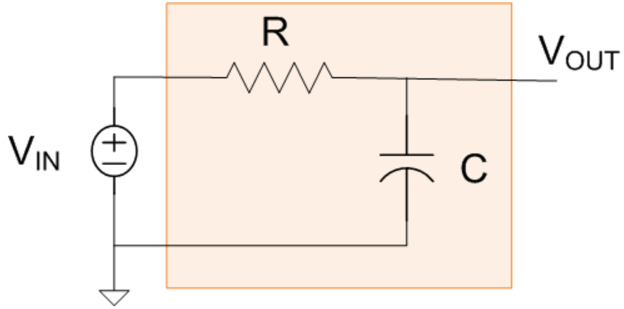
But R_{RAN} and C_{RAN} are approximately R_{PROC} and C_{PROC}

$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{\sigma^2_{\frac{R_{PROC}}{R_{NOM}}} + \sigma^2_{\frac{C_{PROC}}{C_{NOM}}}}$$

recall

$$\sigma_{\frac{R_{PROC}}{R_{NOM}}} = 0.2 \qquad \sigma_{\frac{C_{PROC}}{C_{NOM}}} = 0.1$$

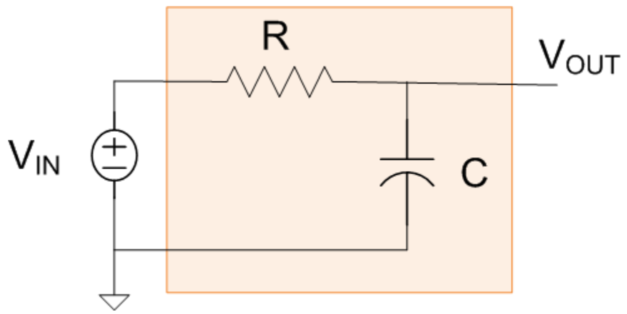
$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{0.2^2 + 0.1^2} = 0.22$$



$$p = \frac{1}{RC}$$

$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{0.2^2 + 0.1^2} = 0.22$$

1. Determine the 3σ range in the pole location
2. Determine the percent of the process lots that will have a pole with mean that is within 10% of the nominal value
3. What can the designer do to tighten the band edge of this filter?



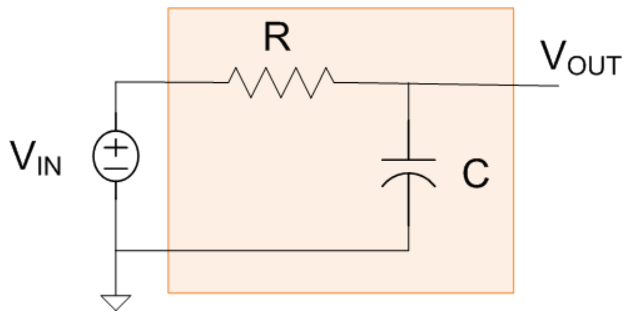
$$p = \frac{1}{RC}$$

$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{0.2^2 + 0.1^2} = 0.22$$

1. Determine the 3σ range in the pole location

The 3σ range is simply $$0.34 \le \frac{p}{p_{NOM}} \le 1.66$$

So, if the nominal pole location is 10KHz, the average value of the pole location from lot to lot will vary between 3.4KHz and 16.6KHz



$$p = \frac{1}{RC}$$

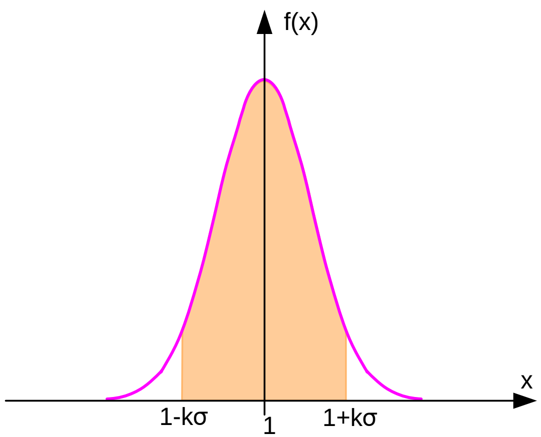
$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{0.2^2 + 0.1^2} = 0.22$$

2. Determine the percent of the process lots that will have a pole with mean that is within 10% of the nominal value

Want probability that $\left\{0.9 < \frac{p}{p_{NOM}} < 1.1\right\}$

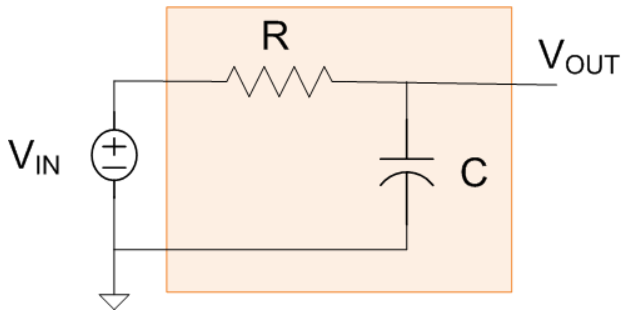
Observe a 10% window is $\left(\frac{.1}{.22}\right)\sigma_{\frac{p}{p_{NOM}}} = 0.45\sigma_{\frac{p}{p_{NOM}}}$

Recall $\frac{p}{p_{NOM}} \sim N\left(1, \sigma_{\frac{p}{p_{NOM}}}\right)$ For a kσ window the probability of being inside that window is the area under the pdf curve between 1– kσ and 1+kσ



Observe

$$\tilde{p} = \frac{\frac{p}{p_{NOM}} - 1}{\sigma_{\frac{p}{p_{NOM}}}} \sim N(0,1)$$



$$p = \frac{1}{RC}$$

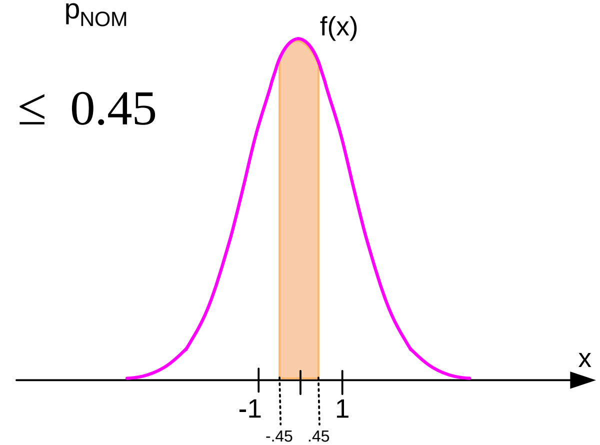
$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{0.2^2 + 0.1^2} = 0.22$$

2. Determine the percent of the process lots that will have a pole with mean that is within 10% of the nominal value

Observe a 10% window is $\left(\frac{.1}{.22}\right)\sigma_{\frac{p}{p_{NOM}}} = 0.45\sigma_{\frac{p}{p_{NOM}}}$

$$1 - 0.45\sigma_{\frac{p}{p_{NOM}}} \le \frac{p}{p_{NOM}} \le 1 + 0.45\sigma_{\frac{p}{p_{NOM}}} \Rightarrow -0.45 \le \tilde{p} \le 0.45$$

$$\tilde{p} = \frac{\frac{p}{p_{NOM}} - 1}{\sigma_{\frac{p}{p_{NOM}}}}$$

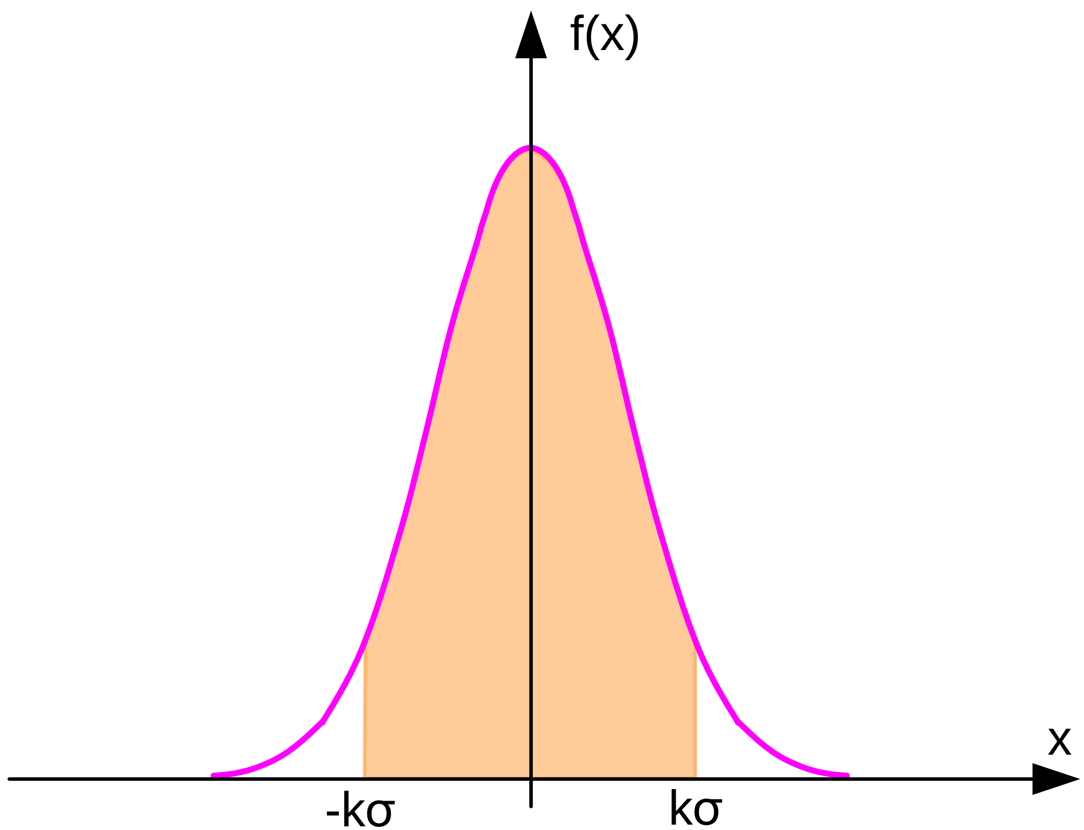


For a Gaussian variable, this area is given by

$$\theta_{prob} = 2F_{N(0,1)}(k) - 1 = 2F_{N(0,1)}(0.45) - 1$$

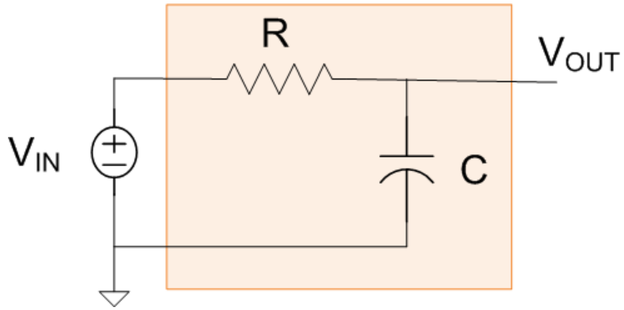
Offset Voltage Distribution

Pdf of zero-mean Gaussian distribution



Percent between:	±σ	68.3%
	±2σ	95.5%
	±3σ	99.73%

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997
4.0	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998	0.99998	0.99998	0.99998



$$p = \frac{1}{RC}$$

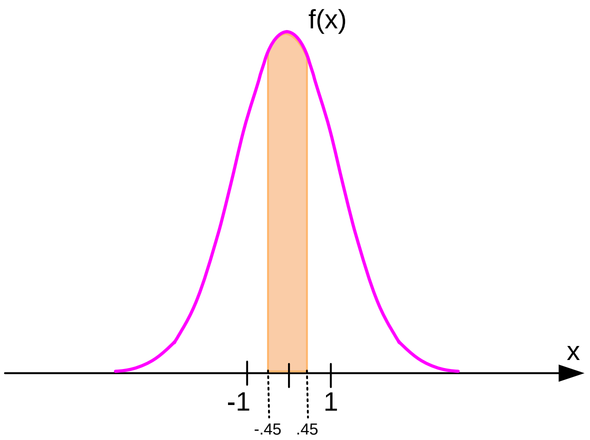
$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{0.2^2 + 0.1^2} = 0.22$$

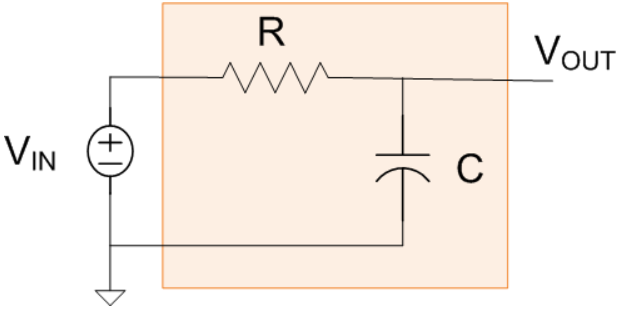
2. Determine the percent of the process lots that will have a pole with mean that is within 10% of the nominal value

$$\theta_{prob} = 2F_{N(0,1)}\left(0.45\right) - 1$$

$$\theta_{prob} = 2 \bullet .6736 - 1 = 0.347$$

Thus, approximately 35% of the wafer lots will have a pole within 10% of the nominal value





$$p = \frac{1}{RC}$$

$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{0.2^2 + 0.1^2} = 0.22$$

3. What can the designer do to tighten the band edge of this filter?

Stay Safe and Stay Healthy !

End of Lecture 14